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CHAMP Dewarp Equations

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1. Derivation

The method is based on idea that the fringe frequency is known and the opd steps in the ABCD are known but not equally spaced. The basic fringe equation is.

$$I_i = I_0 \left[1 + V \cos(2\pi k(\mathrm{opd}_i - \mathrm{opd}_0) - \phi_0)\right] + I_n \tag{1}$$

where opd_i is the current opd, opd_0 is the group delay, V the fringe visibility (including instrumental and atmospheric effects), ϕ_0 is a phase offset, and I_n the noise.

The detector integrates this between the opds opd_i and opd_{i+1} over the spectral wavenumbers k_l to k_h. We note $k_0 = (k_l + k_h)/2$ the central wavenumber, $x_i = (\mathrm{opd}_{i+1} + \mathrm{opd}_i)/2$ the effective opd at frame i, $\Delta\mathrm{opd}_i = (\mathrm{opd}_{i+1} - \mathrm{opd}_i)$ the differential opd, $\Delta k = (k_h - k_l)$ the spectral bandwidth.

The phase tracking estimator searches for the phase ϕ_0. The true fringe intensity is then:

$$I_i = \Delta k \Delta\mathrm{opd}_i \left\{ I_0 \left(1 + V \,\mathrm{sinc}\left[\pi \Delta k (x_i - \mathrm{opd}_0)\right] \mathrm{sinc}\left[\pi k_0 \Delta\mathrm{opd}_i\right] \cos\left[2\pi k_0 (x_i - \mathrm{opd}_0) - \phi_0\right]\right) + I_n \right\} \tag{2}$$

To keep the notation simple, the global factor $\Delta k \Delta\mathrm{opd}_i$ is from now on put into the I_0 and I_n terms, so that we can rewrite:

$$I_i = I_n + I_0 + A_0 \cos\left[2\pi k_0 (x_i - \mathrm{opd}_0) - \phi_0\right] \tag{3}$$

where A_0 contains the sinc functions and the instrumental visibility. The true quadrature terms X,Y, and total flux N (for n reads) will be defined:

$$N = \sum_i I_0 = n I_0 \tag{4}$$

$$X = A_0 \sin(\phi_0) \tag{5}$$

$$Y = A_0 \cos(\phi_0), \tag{6}$$

The fringe squared visibility (which includes the squared sinc functions) is given by:

$$V_{\mathrm{tot}}^2 = \left(\frac{A_0}{I_0}\right)^2 = \frac{n^2}{N^2}(X^2 + Y^2) \tag{7}$$

This may be used to retrieve the target visibility or find the group-delay position (as it will be maximized for $x_i = \mathrm{opd}_0$). More importantly, the phase is given by:

$$\phi_0 = \mathrm{atan}\left(\frac{X}{Y}\right) \tag{8}$$

What we measure for arbitary scanning using a numerical fourier transform is X_m, Y_m, and N_m:

$$N_m = \sum_{i=1}^{n} I_i \tag{9}$$

$$X_m = \sum_{i=1}^{n} I_i \cos(2\pi k_0 x_i) \tag{10}$$

$$Y_m = \sum_{i=1}^{n} I_i \sin(2\pi k_0 x_i), \tag{11}$$

Now we need two assumptions. First that the background is substracted, so that $I_n \simeq 0$. Second, that the group delay has been estimated and removed, so that we can drop the terms opd_0.

So the goal is to find X_0,Y_0, and N_0 for a given x_i,k_0 and measured X_m, Y_m, N_m, and nreads n. We have:

$$I_i = I_0 + A_0 \cos(2\pi k_0 x_i - \phi_0) \tag{12}$$

$$= \frac{N_0}{n} + X_0 \sin(2\pi k_0 x_i) + Y_0 \cos(2\pi k_0 x_i) \tag{13}$$

$$\begin{pmatrix} N_m \\ X_m \\ Y_m \end{pmatrix} = \begin{bmatrix} 1 & \sum_S & \sum_C \\ \sum_C /n & \sum_{SC} & \sum_{CC} \\ \sum_S /n & \sum_{SS} & \sum_{SC} \end{bmatrix} \begin{pmatrix} N_0 \\ X_0 \\ Y_0 \end{pmatrix} \tag{14}$$

where:

$$\sum_S = \sum_{i=1}^{n} \sin(2\pi k_0 x_i) \tag{15}$$

$$\sum_C = \sum_{i=1}^{n} \cos(2\pi k_0 x_i) \tag{16}$$

$$\sum_{SC} = \sum_{i=1}^{n} \sin(2\pi k_0 x_i) \cos(2\pi k_0 x_i) \tag{17}$$

$$\sum_{SS} = \sum_{i=1}^{n} \sin(2\pi k_0 x_i) \sin(2\pi k_0 x_i) \tag{18}$$

$$\sum_{CC} = \sum_{i=1}^{n} \cos(2\pi k_0 x_i) \cos(2\pi k_0 x_i) \tag{19}$$

If we want to include the spectral bandwidth term in the dewarping (this gives a better instantaneous visibility and phase estimation, but then the resulting visibility cannot be used to determine the group delay) we get this matricial equation:

$$\begin{pmatrix} N_m \\ X_m \\ Y_m \end{pmatrix} = \begin{bmatrix} 1 & \sum_{CB} & \sum_{SB} \\ \sum_C /n & \sum_{CCB} & \sum_{SCB} \\ \sum_S /n & \sum_{SCB} & \sum_{SSB} \end{bmatrix} \begin{pmatrix} N_0 \\ X_0 \\ Y_0 \end{pmatrix} \tag{20}$$

where we have used the shorthand for pre-calculated constants:

$$\sum_{SB} = \sum_{i=1}^{n} \mathrm{sinc}(\pi \Delta k x_i) \sin(2\pi k_0 x_i) \tag{21}$$

$$\sum_{CB} = \sum_{i=1}^{n} \mathrm{sinc}(\pi \Delta k x_i) \cos(2\pi k_0 x_i) \tag{22}$$

$$\sum_{SCB} = \sum_{i=1}^{n} \mathrm{sinc}(\pi \Delta k x_i) \sin(2\pi k_0 x_i) \cos(2\pi k_0 x_i) \tag{23}$$

$$\sum_{SSB} = \sum_{i=1}^{n} \mathrm{sinc}(\pi \Delta k x_i) \sin(2\pi k_0 x_i) \sin(2\pi k_0 x_i) \tag{24}$$

$$\sum_{CCB} = \sum_{i=1}^{n} \mathrm{sinc}(\pi \Delta k x_i) \cos(2\pi k_0 x_i) \cos(2\pi k_0 x_i) \tag{25}$$

The matrix should be invertible and its determinant non null (need to double check this).

2. Implementation Notes

2.1. JDM notes

Note that in the inital CHAMP code from 2009, there is a simplified bias correction that simply subtracts the X_{bias}, Y_{bias} from each X,Y based on a measurement without fringes. This has a dramatically good effect and is equivalent to using of the 2 of the 3 terms in the above equqtions [the last term is the so-called bias term which comes from the fact that the sum of cosine of sum of sine is not zero for a warped stroke. However, this does not take into account the 'cross-terms' that can feed Y power into X for warped strokes. For a phase servo around a constant phase target, this seem to be adequate for we will probably need a more accurate estimator if we wish to use the amplitude information for group delay.

All the $\sum_*$ terms can be pre-computed and most of these already are in the current code once the opds and wavelengths are defined. Note these are pre-computed but not integrated into final values since this depends on number of pixels in the fringe window which can be adjusted.

So the way I see this is that the code is already collecting X_m, Y_m, N_m, and all the $\sum$ terms (which don't change). Then one just has to write a routine that does the inversion of the 3 simultaneous equations. This is slightly tricker than it sounds but should be easy enough through a 3x3 matrix inversion. This will then provide an estimate of the “true” X_0, Y_0, N_0, and of course the amplitude and phase of the fringe derive from this. This still needs checking though that this will actually work.

I have not worked on how to do noise debias when one does averaging .. This might be necessary to do when one is tring to signal to noise estimate of accumulated power spectra for instance.

2.2. FB notes

I’ve included the bandwidth smearing into the equations and added the matrix inversion. The place where the pre-computation of the sums can be done is difficult to chose, but the sin, cos and sinc can be pre-computed as soon as the opds are known.