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CHAMP Dewar Equations

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1. Derivation

The method is based on idea that the fringe frequency is known and the opd steps in the ABCD are known but not equally spaced.

So assume we have a known fringe of spatial frequency $k_0 = \frac{2\pi}{\lambda_0}$ and true phase ϕ_0. Then the true fringe intensity for a known opd x_i:

$$I_i = A_0 sin(2\pi k_0 x_i - \phi_0) + I_0 \tag{1}$$

The true quadrature terms X,Y, and total flux N (for n reads) will be defined:

$$X_0 = -A_0 sin(\phi_0) \tag{2}$$

$$Y_0 = A_0 cos(\phi_0) \tag{3}$$

$$N_0 = \Sigma_i I_0 = nI_0, \tag{4}$$

What we measure for arbitary scanning using a numerical fourier transform is X_m, Y_m, and N_m:

$$X_m = \Sigma_{i=1}^{n} I_i cos(2\pi k_0 x_i) \tag{5}$$

$$Y_m = \Sigma_{i=1}^{n} I_i sin(2\pi k_0 x_i) \tag{6}$$

$$N_m = \Sigma_{i=1}^{n} I_i \tag{7}$$

So the goal is to find X_0,Y_0, and N_0 for a given x_i,k_0 and measured X_m, Y_m, N_m, nreads n I can come up with the 3 simulateous equations for this but still need another program to do the inversion.

Here is my intermediate result after plugging into the above equations.

$$X_m = Y_0\Sigma_{SC} + X_0\Sigma_{CC} + N_0\Sigma_C/n \quad (8)$$
$$Y_m = Y_0\Sigma_{SS} + X_0\Sigma_{SC} + N_0\Sigma_S/n \quad (9)$$
$$N_m = Y_0\Sigma_S + X_0\Sigma_C + N_0 \quad (10)$$

where I have used the shorthand for pre-calculated constants:

$$\Sigma_S = \Sigma_{i=1}^{n} sin(2\pi k_0 x_i) \quad (11)$$
$$\Sigma_C = \Sigma_{i=1}^{n} cos(2\pi k_0 x_i) \quad (12)$$
$$\Sigma_{SC} = \Sigma_{i=1}^{n} sin(2\pi k_0 x_i) cos(2\pi k_0 x_i) \quad (13)$$
$$\Sigma_{SS} = \Sigma_{i=1}^{n} sin(2\pi k_0 x_i) sin(2\pi k_0 x_i) \quad (14)$$
$$\Sigma_{CC} = \Sigma_{i=1}^{n} cos(2\pi k_0 x_i) cos(2\pi k_0 x_i) \quad (15)$$
$$(16)$$

the above 3 equations should be straightforward to solve numerically.

2. Implementation Notes

Note that in the inital CHAMP code from 2009, there is a simplified bias correction that simply subtracts the X_{bias}, Y_{bias} from each X,Y based on a measurement without fringes. This has a dramatically good effect and is equivalent to using of the 2 of the 3 terms in the above equqtions [the last term is the so-called bias term which comes from the fact that the sum of cosine of sum of sine is not zero for a warped stroke. However, this does not take into account the 'cross-terms' that can feed Y power into X for warped strokes. For a phase servo around a constant phase target, this seem to be adequate for we will probably need a more accurate estimator if we wish to use the amplitude information for group delay.

All the Σ_* terms can be pre-computed and most of these already are in the current code once the opds and wavelengths are defined. Note these are pre-computed but not integrated into final values since this depends on number of pixels in the fringe window which can be adjusted.

So the way I see this is that the code is already collecting X_m, Y_m, N_m, and all the Σ terms (which don't change). Then one just has to write a routine that does the inversion of the 3 simultaneous equations. This is slightly tricker than it sounds but should be easy enough through a 3x3 matrix inversion. This will then provide an estimate of the "true" X_0, Y_0, N_0, and of course the amplitude and phase of the fringe derive from this. This still needs checking though that this will actually work.

I have not worked on how to do noise debias when one does averaging .. This might be necessary to do when one is tring to signal to noise estimate of accumulated power spectra for instance.